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Flutter of a cantilevered elastic and viscoelastic strip☆

V.V. Pokazeyev

Moscow, Russia

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ABSTRACT

The transient panel flutter of a cantilevered elastic and viscoelastic strip, with one end of the strip rigidly fixed and the second end free, is investigated. It is assumed that the flow velocity vector is parallel to the plane of the strip and, with its edges, makes an angle that can take arbitrary positive and negative values. Approximate estimates of the critical flutter velocity are obtained by approximating the solution by special polynomials, by a Laplace transform with respect to time and by Bubnov's method.

In the first investigations of the vibration and stability of a cantilevered strip, assuming motion of the strip in a gas flow in the direction from the fixed end to the free end, the effect of divergence (cylindrical bending of the strip when a certain critical flow velocity is reached) was discovered.[1] Later, approximate values of the critical flutter and divergence velocity were obtained for a cantilevered strip under conditions where the flow velocity vector was directed parallel to the plane of the strip and perpendicular to its edges.[2] In investigations of longitudinal flow along a cantilevered strip, the critical flutter velocity and the value of the corresponding wave formation parameter were calculated for the case where the flow velocity vector was parallel to the plane and edges of the strip.[†]

Below, an approximate solution of the problem of the flutter of a cantilevered strip under conditions where the flow velocity vector is parallel to its plane and, with the edges, makes an angle that can take arbitrary values in the range $[-\pi/2, \pi/2]$ is constructed for the first time. In all cases the approximate solution is based on linear combinations of polynomials that identically satisfy the boundary conditions.

1. Formulation of the problem

We will first consider an infinite elastic strip that, in a rectangular system of coordinates, occupies the region $0 \le y \le l$, $|x| \le \infty$. It is assumed that one end of the strip ($y=0$) is rigidly fixed, while the other ($y=l$) is free. Gas flows around the strip with a velocity vector $\boldsymbol{\upsilon} = \mathbf{n}_0\upsilon$, $\mathbf{n}_0 = (\cos\theta, \sin\theta)$, and with the following fixed parameters: pressure p_0, density ρ_0 and sound velocity c_0.

Under conditions where the excess pressure in the gas flow is defined by piston theory formulae, the vibrations of the strip are described by the equation[3]

$$D_0\Delta^2 w + \rho h\frac{\partial^2 w}{\partial t} + \frac{\gamma p_0}{c_0}\left(\frac{\partial w}{\partial t} + \upsilon\mathbf{n}_0\cdot\operatorname{grad} w\right) = 0, \quad D_0 = \frac{E_0 h^3}{12(1-\nu^2)} \tag{1.1}$$

where D_0 is the cylindrical stiffness, E_0 is Young's modulus, ρ and ν are the density and Poisson's ratio of the strip material, and γ is the polytropic exponent of the gas.

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E-mail address: pokazeyev@yandex.ru.

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Equation (1.1) is investigated under the cantilever boundary conditions

$$y = 0: \quad w = 0, \quad \frac{\partial w}{\partial y} = 0$$

$$y = l: \quad \frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2} = 0, \quad \frac{\partial^3 w}{\partial y^3} + (2-\nu)\frac{\partial^3 w}{\partial x^2 \partial y} = 0 \tag{1.2}$$

and with the initial data determined by the type of perturbation.

The problem consists of determining the lowest value of the flow velocity υ_* such that when $\upsilon < \upsilon_*$ the perturbed motion will be asymptotically stable, and when $\upsilon > \upsilon_*$ it will be asymptotically unstable.

We will introduce the dimensionless coordinates x/l and y/l and the velocity $M = \upsilon/c_0$ into Eqs. (1.1), retaining the previous notation for the coordinates. In dimensionless coordinates, Eq. (1.1) takes the form

$$\Delta^2 w + a_1\frac{\partial w}{\partial t} + a_2\frac{\partial^2 w}{\partial t^2} + a_3 M\mathbf{n}_0 \cdot \operatorname{grad} w = 0 \tag{1.3}$$

where the following notation is introduced

$$a_1 = 12(1-\nu^2)l^4\gamma p_0/(h^3 E_0 c_0), \quad a_2 = 12(1-\nu^2)l^4\rho/(h^3 E_0), \quad a_3 = 12(1-\nu^2)l^3\gamma p_0/(h^3 E_0)$$

2. The structure of the approximate solution

We will choose the initial perturbation, bounded at infinity, in the form

$$t = 0: \quad w(x, y, 0) = \sum_k G_{k,0}\varphi_k(y)\exp(-i\alpha x),$$

$$w'_t(x, y, 0) = \sum_k G_{k,1}\varphi_k(y)\exp(-i\alpha x), \quad \alpha, G_{k,0}, G_{k,1} \in R$$

Accordingly, for the deflection we will adopt the representation

$$w = \sum_k G_k(t)\varphi_k(y)\exp(-i\alpha x)$$

in which each of the functions $\varphi_k(y)$ satisfies the conditions

$$\varphi_k(0) = 0, \quad \varphi'_k(0) = 0, \quad \varphi''_k(1) - \nu\alpha^2\varphi_k(1) = 0, \quad \varphi'''_k(1) - (2-\nu)\alpha^2\varphi'_k(1) = 0 \tag{2.1}$$

It is convenient to choose as $\varphi_k(y)$ the set of polynomials

$$\varphi_k(y) = A_k y^k + B_k y^{k-1} + C_k y^{k-2}, \quad k \geq 4 \tag{2.2}$$

imposing on each of the functions $\varphi_k(y)$ the boundary conditions. After substituting (2.2) into (2.1), we arrive at a system of linear equations for determining the coefficients A_k, B_k and C_k:

$$\zeta_k A_k + \zeta_{k-1}B_k = -\zeta_{k-2}C_k, \quad k\xi_k A_k + (k-1)\xi_{k-2}B_k = -(k-2)\xi_{k-3}C_k \tag{2.3}$$

where

$$\zeta_k = k(k-1) - \alpha^2\nu, \quad \xi_k = k(k-1) - \alpha^2(2-\nu)$$

It is not difficult to show that the determinant of system (2.3) is non-zero for any $\nu \in [0, 0.5]$ and integers $k \geq 4$. Since the coefficient C_k can be specified in an arbitrary way, we will assume that $C_k = 1/k!$, and then, from system (2.3), we will obtain

$$A_k = ((k-1)\zeta_{k-2}\xi_{k-2} - (k-2)\zeta_{k-1}\xi_{k-3})/D_k, \quad B_k = -(k\zeta_{k-2}\xi_{k-1} - (k-2)\zeta_k\xi_{k-3})/D_k$$

$$D_k = k!(k\zeta_{k-1}\xi_{k-1} - (k-1)\zeta_k\xi_{k-2})$$

3. Investigation of the stability of the solution

For the bending of the strip we will use the representation

$$w = \sum_{k=4}^{n} G_k(t)\varphi_k\exp(-i\alpha x), \quad \varphi_k = \varphi_k(y) \tag{3.1}$$

We substitute expressions (3.1) into Eq. (1.3) and apply a Laplace transformation with respect to time, taking into account the initial conditions (s is the transformation parameter). As a result, we obtain the equation

$$\sum_{k=4}^{n} \tilde{G}_k(s)\Phi_k(y, \alpha, \theta, M, s) = Q(s, y)$$

$$\Phi_k(\alpha, \theta, M, s, y) = \varphi_k^{(4)} - 2\alpha^2\varphi_k'' + \alpha^4\varphi_k + (a_2 s^2 + a_1 s)\varphi_k + a_3 M(\varphi_k' \sin\theta - i\alpha\varphi_k\cos\theta) \tag{3.2}$$

in which the function $Q(x, y)$ depends only on the initial data and the coefficients of the problem.

We multiply relation (3.2) by $\varphi_m(y)$ ($m = 4, 5, \ldots, n$) and integrate over the segment [0, 1]; we obtain a system of linear equations in the unknown images $\tilde{G}_k(s)$

$$\sum_{k=4}^{n} \tilde{G}_k(s)\int_0^1 \Phi_k(\alpha, \theta, M, s, y)\varphi_m(y)dy = \int_0^1 Q(s, y)\varphi_m(y)dy, \quad m = 4, 5, \ldots, n \tag{3.3}$$

The behaviour of the originals $G_k(t)$ and the forms of motion of the strip depend on the zeros of the determinant of system (3.3), which comprises a polynomial $P_r(s)$ of power $r = 2n - 6$. The vibration of the strip will be asymptotically stable (decrease exponentially) if all the complex roots of the polynomial are located in the left-hand half-plane. If any of the roots of the polynomial transfer into the right-hand half-plane, the motion of the strip becomes asymptotically unstable. A case corresponding to the boundary of the regions of stability and instability is one where, for one of the roots, the requirement $\mathrm{Re}\, s_j = 0$ is satisfied, provided all remaining roots are located in the left-hand half-plane. The flutter velocity $\tilde{M}$ is related to these conditions. It depends on the wave-formation parameter α: by definition, for the critical flutter velocity we assume that $M_* = \tilde{M}(\alpha_*)$, and α_* is found from the condition for the function $\tilde{M}(\alpha)$ to be a minimum.

4. An elastic strip

In longitudinal and transverse flow around the elastic strip, the behaviour of the approximate solutions containing polynomials up to and including the ninth power was investigated. Specific calculations were carried out for the following values of the parameters

$$p_0/E_0 = 5\times10^{-7},\ \rho = 8\times10^3\ \mathrm{kg/m^3},\ \gamma = 1.4,\ \nu = 0.3,\ c_0 = 330\ \mathrm{m/s},\ l/h = 250$$

The values of the critical flutter velocity M_* for longitudinal and transverse flow around the strip for various powers of the polynomial n are given in Table 1.

The calculation results indicate good convergence of the approximation process. When $\theta = 0$ and $\theta = \pi/2$, the values of the critical flutter velocity are practically identical with those obtained earlier.[2] In Kudryavtsev's dissertation, for the given critical flutter velocity and the corresponding wave-formation parameter for longitudinal flow, the values $A = a_3 M_* \approx 7.83$ and $\alpha_* \approx 1.88$ are obtained; according to the results of the present paper, $A \approx 7.79$ and $\alpha_* \approx 1.92$.

When $\theta = -\pi/2$, when the flow velocity increases, the root $s = 0$ is the first to fall on the imaginary axis, while minimization of the flutter velocity with respect to the wave-formation parameter leads to the value $\alpha = 0$. Here, the critical velocity in terms of the reduced velocity gives the value $A = a_3 M_* \approx 6.33$, which is in agreement with the result obtained earlier[1] for the divergence.

Under conditions where the flow velocity vector makes an angle θ with the edges of the strip, for the deflection an approximation was chosen that contained a sixth-order polynomial:

$$w = \sum_{k=4}^{6} G_k(t)\varphi_k(y)\exp(-i\alpha x) \tag{4.1}$$

The results of calculations are given below:

θ/π	–1/2	–7/16	–5/18	–1/4	–1/6	–1/18	–1/36
α_*	0	0	0	0.87	1.50	1.81	1.87
M_*	0.0530	0.0540	0.0692	0.0717	0.0674	0.0643	0.0646

θ/π	0	1/18	1/4	7/16	15/32	31/64	1/2
α_*	1.92	2.01	2.38	3.44	0.042	0.021	0
M_*	0.0652	0.0680	0.1043	0.4901	1.1480	1.1440	1.1427

An analysis of the results of the calculations enables the following conclusions to be drawn.

Table 1

n	$\theta = 0$, $\alpha_* = 1.92$	$\theta = \pi/2$, $\alpha_* = 0$	$\theta = -\pi/2$, $\alpha_* = 0$
5	0.06521	1.06848	0.05223
6	0.06520	1.14272	0.05300
7	0.06520	1.15895	0.05300
9	0.06520	1.15817	0.05300

1. When $\theta \in [0, \pi/2]$, as in the case of cantilevered strip, close to $\theta = \pi/2$ an angle θ_0 ($7\pi/16 < \theta_0 < 15\pi/32$) exists, in the neighbourhood of which the nature of the vibration of the strip changes. When $\theta \in [0, \theta_0]$, when θ increases there is an increase in the values of the critical velocity and in the corresponding values of the wave-formation parameter. When $\theta \in [\theta_0, \pi/2]$ there is a small reduction in the values of M_*, and here the parameter α_* falls sharply to zero, with a sudden change at the point θ_0.
2. When the values of θ decrease from 0 to $-\pi/2$, the critical flutter velocity initially decreases slightly, and then increases slightly to a value corresponding to the angle $\theta_1 \in (-5\pi/18, -\pi/4)$. When $\theta < \theta_1$, flutter gives way to divergence.
3. Further, when $\theta \in [-\pi/2, \theta_1]$, minimization of the critical velocity with respect to α leads to the value $\alpha = 0$, the root $s = 0$ of polynomial $P_r(s)$ is the first to fall on the imaginary axis, and for the critical velocity and each angle $\theta \in [-\pi/2, \theta_1]$ the relation $M_*(\theta)| \sin\theta| = M_{\text{div}}$ is satisfied, where M_{div} is the critical velocity of divergence when $\theta = -\pi/2$.

Thus, the lowest value of the critical flutter velocity is reached in the case of longitudinal or near-longitudinal flow (for small negative values of $\theta > -\pi/18$) along the cantilevered strip. The divergent state and, together with it, cylindrical bending arise at θ values considerably greater than $-\pi/2$.

The fact that the M_* values obtained above do not exceed unity, or are close to unity, does not play a special role – when the ratio l/h decreases the values of the critical velocity increase, and here the general pattern of vibration found is retained, and the principal conclusions remain unchanged.

5. A viscoelastic strip

We will assume that the material of the strip is linear viscoelastic:

$$\sigma(t) = E_0\left(\varepsilon(t) - \varepsilon_1\int_0^t \Gamma(t-\tau)\varepsilon(\tau)d\tau\right) \equiv E_0(1-\varepsilon_1\hat{\Gamma}_1)\varepsilon(t)$$

We will write the equation of vibrations in the form[4]

$$D_0(1-\varepsilon_1\hat{\Gamma}_1)\Delta^2 w + \rho h\frac{\partial^2 w}{\partial t^2} + \frac{\gamma p_0}{c_0}\left(\frac{\partial w}{\partial t} + \upsilon\mathbf{n}_0\cdot\operatorname{grad}w\right) = 0 \tag{5.1}$$

The boundary conditions (1.2) remain as before.

For the deflection w we choose approximation (4.1), substitute this expression into (5.1) and take a Laplace transformation of the result. We obtain the equation

$$\sum_{k=4}^{6}\tilde{G}_k(s)(1-\varepsilon_1\hat{\Gamma}_1(s))(\varphi_k^{(4)} - 2\alpha^2\varphi_k'' + \alpha^4\varphi_k) + (a_2 s^2 + a_1 s)\varphi_k +$$
$$+ a_3 M(\varphi_k'\sin\theta - i\alpha\varphi_k\cos\theta) = Q(s, y), \quad \varphi_k = \varphi_k(y)$$

The entire subsequent procedure of transformations and investigations of the stability of the solution are largely identical with those in the previous section.

In the calculations it was initially assumed that the relaxation kernel contains one exponential term $\Gamma(t) = \exp(-\beta_1 t)$. Then, the determinant of the corresponding system of linear equations can be written in the form $\Delta = R_9(s)/(s+\beta_1)^3$, where $R_9(s)$ is a ninth-order polynomial, and subsequent investigation of the stability depends on the motion of the roots of this polynomial.

The calculation results for the case when $\varepsilon_1 = 10^{-2}$, $\beta_1 = 10^{-1}$ and $l/h = 250$ are given below:

θ/π	−1/2	−7/16	−5/18	−1/4	−7/30	−1/6	−1/18
α_0	0	0	0	0.87	1.09	1.50	1.81
M_0	0.05300	0.05404	0.06919	0.07165	0.07117	0.06736	0.06431
α_*	0	0	0	0	1.09	1.50	1.81
M_*	0.04770	0.04863	0.06227	0.06746	0.07119	0.06738	0.06432

θ/π	0	1/18	1/4	7/16	15/32	31/64	1/2
α_0	1.92	2.01	2.38	3.44	0.042	0.021	0
M_0	0.06520	0.06795	0.10428	0.49005	1.14798	1.14403	1.14272
α_*	1.92	2.01	2.39	3.44	0.042	0.021	0
M_*	0.06521	0.06797	0.10430	0.49012	1.14799	1.14403	1.14272

where M_0 is the critical flutter velocity, determined from the instantaneous modulus, α_0 is the corresponding value of the wave-formation parameter, and M_* and α_* are values of the critical flutter velocity and the wave-formation parameter for a viscoelastic strip.

These results enable the following conclusions to be drawn: when $\theta \in [-7\pi/30, \pi/2]$ an analogy is observed with the results obtained for a hinged viscoelastic strip,[5] the critical flutter velocity of the viscoelastic strip is practically identical with the critical velocity calculated from the instantaneous modulus, and here the corresponding values of the wave-formation parameter are similar or identical.

Investigation of the forms of motion indicates that, when $M = 0$, of the nine roots of the polynomial $R_9(s)$, six roots are pairwise conjugate with identical real parts $\operatorname{Re} s_i \approx -a_1/(2a_2) \approx -26.52$ ($i = 1, \ldots, 6$) and different imaginary parts. Another three roots are real and roughly the same: $s_j \approx -0.0899$ ($j = 7, 8, 9$). When the values of M increase, the given six conjugate roots become quasi-conjugate, and one of these roots, moving along a trajectory very similar to the trajectory of motion of the corresponding root in the elastic case, intersects the imaginary axis in roughly the same region.

When M increases, the three real roots become complex, their real parts decrease very slightly and, when the critical flutter velocity is reached, are displaced to the value -0.0900 and the imaginary parts of these roots vary in the range $(10^{-6}, 10^{-3})$.

An entirely different picture is observed in the case of divergence. We note first of all that the problem of flow along a cantilevered viscoelastic strip at an angle $\theta=-\pi/2$ allows of a more detailed investigation. Thus, assuming that, in general form, the following representation holds for the deflection

$$w = A(t)\psi(y)\exp(-i\alpha x) \tag{5.2}$$

where $A(t)$ and $\psi(y)$ are unknown functions of time and coordinate, we substitute expression (5.2) into Eq. (5.1) and carry out a Laplace transformation. Then, for one exponential term in the relaxation kernel, we obtain the equation

$$\tilde{A}(s)\Psi(y, s, \alpha, \varepsilon_1, \beta_1, M) = (s+\beta_1)Q(s, y)$$

where

$$\Psi(y, s, \alpha, \varepsilon_1, \beta_1, M) = (s+\beta_1-\varepsilon_1)(\psi^{(4)}-2\alpha^2\psi''+\alpha^4\psi)+(a_2s^2+a_1s)\psi+$$
$$+(s+\beta_1)a_3M(\psi'\sin\theta-i\alpha\psi\cos\theta), \quad \psi = \psi(y)$$

Assuming $s=0$ in the equation

$$\Psi(y, s, \alpha, \varepsilon_1, \beta_1, M) = 0$$

we arrive at the relation

$$\psi^{(4)}-2\alpha^2\psi''+\alpha^4\psi-a_3M_1\psi' = 0, \quad M_1 = M/(1-\lambda_1), \quad \lambda_1 = \varepsilon_1/\beta_1$$

and, under conditions where $\alpha=0$, we obtain an equation of the form

$$\psi^{(4)}-a_3M_1\psi' = 0$$

which, apart from the notation, repeats the corresponding equation for a cantilevered elastic strip.[1] Thus, if $M_{1,\mathrm{div}}$ is the value of the critical divergence velocity of an elastic strip at $\theta=-\pi/2$, then the critical divergence velocity M_* of a viscoelastic strip is defined by the equation $M_*=(1-\lambda_1)M_{1,\mathrm{div}}$, which corresponds to the limiting modulus. Approximate calculations largely confirm this conclusion, and consequently, when $\theta=-\pi/2$, the critical velocity can be calculated as the limiting-modulus critical velocity.

Note that this conclusion holds for relaxation kernels that contain a finite number of exponential terms. For example, for the case where the relaxation kernel contains four exponential terms with the parameters

$$\varepsilon_1 = 2\times10^{-3} = \beta_1/10, \quad \varepsilon_2 = 3\times10^{-3} = \beta_2/10, \quad \varepsilon_3 = 5\times10^{-3} = \beta_3/10, \quad \varepsilon_4 = 10^{-2} = \beta_4/10$$

the results of calculations in the case of longitudinal and transverse flow around the strip are given below (the previous qualitative characteristics of the results are retained):

α_*	1.92	0	0
M_0	0.06520	1.14272	0.05300
M_*	0.06522	1.14272	0.03180

Since for a viscoelastic strip the critical divergence velocity at $\theta=-\pi/2$ is lower than the corresponding critical velocity in the elastic case, it follows that the "divergent" state with cylindrical bending sets in earlier, i.e., at larger negative angles, than in the elastic case. Thus, when $\theta=-\pi/4$, in the case of a cantilevered elastic strip a state of flutter is observed, whereas in the case of a cantilevered viscoelastic strip a state of divergence is observed. Note that the forms of motion of a cantilevered viscoelastic strip when reaching "divergent" states likewise differ from those described earlier. Now, when M increases, one of the three roots that, with $M=0$, were positioned on the real axis, is the first to fall on the imaginary axis.

6. Conclusions

The solution of the problem of the flutter of a cantilevered strip in the form of a linear combination of specially constructed polynomials, which is convenient in calculations, yields a result with sufficient accuracy. It was shown that, for any non-negative angles of flow, as the flow velocity increases there is instability in the form of the flutter with a characteristic travelling wave along the strip. For negative angles of flow, either flutter or divergence is observed. Here, a fundamentally new mechanical effect was found: an entire sector of directions (from $-\pi/4$ to $-\pi/2$) exists for which increase in the flow velocity leads to a divergent state.

For a cantilevered viscoelastic strip, approximate values of the critical flutter velocity were found, on the assumption that the material of the strip is linear viscoelastic, and the relaxation kernel contains exponential terms. Here, the flutter values of the critical velocity are similar to the corresponding values of the elastic problem with an instantaneous Young's modulus, whereas in the case of divergence the values of the critical velocity can differ considerably from the corresponding values for an elastic strip.

Under conditions when the flow velocity vector makes an angle of $-\pi/2$ with the edges of the strip, i.e., it is directed from the free edge of the strip to the fixed end, it has been proved theoretically that the critical divergence velocity is equal to the limiting-modulus critical velocity for a viscoelastic strip.

Finally, note that the equation of vibration of a strip in the present paper is based on the formula from piston theory for the pressure of aerodynamic interaction Δp. At the same time, it is well known that the applicability of piston theory is open to question at low supersonic flow velocities. A number of studies have recently appeared in which the use of a "non-piston" approach in investigating the flutter problem has led to essentially new results; above all, Refs 6 and 7 must be mentioned, and also Refs 8 and 9. Nevertheless, from our point of view, calculations using the piston theory make sense when they are being conducted for the first time.

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